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ABSTRACT

Word problems form a central component in the learning and teaching of mathematics. This paper reports on a study of mathematics teachers' thinking in the teaching of word problems with particular focus on what exemplary high school mathematics teachers value in framing their teaching of word problems and what characterizes their teaching of word problems. Case studies of six teachers were conducted. Data collected through interviews and classroom observations were analyzed for attributes of the participants' thinking and actions that were characteristic of their perspective of teaching word problems. The findings provide insights of teachers' way of thinking and classroom behaviors that are important to motivate and engage students meaningfully in doing word problems. There are five student characteristics the teachers intentionally incorporate in their teaching and eight teaching characteristics they value that form the essence of their teaching of word problems. The findings have implications for what we consider and how we work with prospective and experienced teachers to help them to develop/change/enhance their teaching of word problems. (Author)

TEACHING WORD PROBLEMS: WHAT HIGH SCHOOL MATHEMATICS TEACHERS VALUE

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Word problems form a central component in the learning and teaching of mathematics. This paper reports on a study of mathematics teachers' thinking in the teaching of word problems with particular focus on what exemplary high school mathematics teachers value in framing their teaching of word problems and what characterizes their teaching of word problems. Case studies of six teachers were conducted. Data collected through interviews and classroom observations were analyzed for attributes of the participants' thinking and actions that were characteristic of their perspective of teaching word problems. The findings provide insights of teachers' way of thinking and classroom behaviors that are important to motivate and engage students meaningfully in doing word problems. There are five student characteristics the teachers intentionally incorporate in their teaching and eight teaching characteristics they value that form the essence of their teaching of word problems. The findings have implications for what we consider and how we work with prospective and experienced teachers to help them to develop/change/enhance their teaching of word problems.

This paper is based on a larger study that is investigating the teaching of mathematical word problems from the perspective of the teacher. The focus here is on the perspective of exemplary high school mathematics teachers, in particular, what they value in framing their teaching of word problems in the classroom, and what characterizes their teaching of word problems.

Background and Theoretical Perspective

Recent reform recommendations in mathematics education, for example, the National Council of Teachers of Mathematics [NCTM] standards and principles (NCTM 1989, 1991, 2000) assign a significant role to problem contexts in developing meaning for mathematics and a problem-solving perspective of teaching and learning mathematics. Implementing such recommendations will necessitate an increase in importance in the use of a range of mathematical word problems, routine or non-routine, in the learning and teaching of mathematics. This is likely to pose increase challenges for both teachers and students. In spite of several studies on problem solving and several theoretical prescriptions on how to become a good problem solver and how to teach problem solving, word problems remain a component of mathematics many students fear and experience increasing difficulty with as they move up the Grades.

Studies on mathematical word problems and problem solving have traditionally focused on the learner to study cognitive and affective factors that aid or hinder his/her performance as a problem solver (e.g., Carpenter, Moser & Debout, 1988; Silver, 1985). Such studies generally identified determinants of problem difficulty,

characteristics of successful and unsuccessful problem solvers, and the relation of meta-cognitive factors to problem solving. They were then used as a basis to design, test and prescribe instruction. In particular, studies on arithmetic and algebraic word problems continue to reflect this trend of focusing on learner-related variables. For example, studies on arithmetic word problems, involving elementary students, (Carey, 1991; Cummings, 1991; Debout, 1990; Feinberg, 1988; Fuson & Willis, 1989; Lewis & Mayer, 1987; Sowder, 1988) have looked at the mathematical and linguistic structure of these problems in relation to the children's performance; factors that affect the difficulty of the problem for children; strategies and methods children use; and the errors children make in their solutions. More recently, Verschaffel, Greer and DeCorte (2000) have looked at students' suspension of sense making in doing word problems. They pointed out that after several years of traditional mathematics instruction, students have developed a tendency to reduce word-problem solving to selecting what they take to be the correct arithmetic operation with the numbers given in the problem, without seriously taking into account their common-sense knowledge and realistic considerations about the problem context. A similar situation has existed for studies on algebraic word problems, involving high school students, where, for example, the focus has been on students' errors and methods in the translation process (Clement, 1982; Crowley, Thomas & Tall, 1994; Kaput & Sims-Knight, 1983; MacGregor & Stacey, 1993; Wollman, 1983). Reed (1999) provides a current overview of studies of both arithmetic and algebraic word problems.

Given the focus on the learner, studies on mathematical problem solving have generally ignored or trivialized the classroom teacher. Even in cases where instruction was considered, the studies were still biased towards the learner in that they generally focused on the effectiveness (in terms of students' performance) of instructional methods designed by the researcher and administered under experimental conditions. Thus, while these studies have enhanced our understanding of important issues associated with the learning of word problems, they offer very little on the teacher. But this situation was eventually considered by some researchers to be a significant limitation in understanding and improving the teaching, and consequently, the learning of problem solving. For example, such lack of research on the teacher in problem-solving instruction has been proposed as a significant limitation in understanding and improving the teaching of problem solving (Lester, 1994; 1985). The initial concern was that "we will make no progress toward developing practical guidelines for mathematics teachers to follow until we are able to make explicit that knowledge about teaching that good teachers have internalized and made 'second nature'" (Lester, 1985, p.56). The study in this paper is intended to make explicit aspects of teacher thinking and classroom behavior that exemplary high school teachers consider important in framing their teaching of word problems in the classroom.

The study, then, is framed in the theoretical perspective of teacher thinking in which teachers are viewed as creating their own meaning to make sense of their teach-

ing, i.e., a constructivist orientation of knowledge construction. Research on teacher thinking is well established (Elbaz, 1991; Fenstermacher, 1994; Grossman, 1990). The importance of researching the teacher is associated with the view that teachers are the determining factor of how the curriculum, mathematics in this case, is taught. Within this humanistic view of the teacher, the teacher's thinking is valued as a basis for understanding her/his classroom behaviors, in particular, and teaching, in general. This validates the importance to learn from teachers what they do and how they make sense of what they do (i.e., their perspective) in the context of the classroom.

In recent years, there has been increased focus on researching the mathematics teacher. Thus, there is a growing body of literature on mathematics teachers' content knowledge, beliefs, conceptions, classroom practices, learning, professional development and change (e.g., Lampert & Ball, 1998; Chapman, 1997; Fennema & Nelson, 1997; Lloyd & Wilson, 1998; Schifter, 1998; Thompson, 1992; Tzur, Simon, Heinz & Kinzel, 2001). These studies have provided us with insights on, for example, the relationship between beliefs and teaching, deficiencies in teachers' content knowledge, and the challenges of teacher education and change. The literature also suggests that mathematics teachers' beliefs and conceptions about mathematics and its teaching and learning are significantly related to their teaching. As Ernest (1989) explained, in relation to mathematics teachers, it is the teachers' mental or espoused models of teaching and learning that are transformed into classroom practices. This view of the teacher suggests that the teaching of word problems in a classroom context could depend on the teachers' underlying beliefs and conceptions framing it, or, more generally, on the teacher's thinking. However, ongoing research to understand the high school mathematics classroom from the teacher's perspective, in general, and explicitly dealing with the teaching of word problems, in particular, is important as we try to reform the teaching of mathematics. In this study, the teachers' perspective includes their conceptions, their intentions, what they value and personal meaning.

Research Method

The study is framed in a naturalistic inquiry perspective that facilitates a phenomenological context to interpret one's thinking and behavior (Taylor & Bogdan, 1998). The participants were six experienced high school teachers (4 females, 2 males, 16-30 years experience), considered to be excellent teachers in their school system. Their students have consistently scored very high on their provincial standardized examination that has a growing emphasis on problem solving following the NCTM standards. They had positive attitudes to teaching mathematics, in general, and word problems, in particular. They have been involved in making presentations at mathematics teachers' conferences and/or leading professional development workshops, although not specifically related to teaching word problems.

The main sources of data for the study were interviews and classroom observations. The interviews were open-ended to give the participants an opportunity to talk

freely and in an unprompted way about their own understandings of teaching word problems. They were also partly framed in a narrative perspective to allow the teachers to describe their experiences as actual phenomena. This produced stories of actual events that embodied the meaning of the participants' behavior. The interviews dealt with the participants' thinking in three contexts: past, present and future. The past dealt with their past experiences with word problems as both students and teachers focusing on teacher and student presage characteristics, task features, and contextual conditions The present dealt with their current practice with particular emphasis on classroom processes, planning and intentions. The future dealt with expectations, e.g., possible changes due to personal or external factors, and, "if ... what" scenarios.

Classroom observations focused on the participants' actual instructional behaviors during lessons involving word problems. Special attention was given to what the teachers and students did during instruction and how their actions interacted. Complete units involving word problems were observed. Post-observation discussions with the participants focused on clarifying their thinking in relation to their actions. All interviews and discussions were audio taped and transcribed. The data were thoroughly reviewed by the researcher and two research assistants working independently to identify attributes (e.g., recurring conceptions/beliefs, intentions, judgements) of the participants' thinking and actions that were characteristic of their perspective of teaching word problems. These attributes were grouped into themes. Both attributes and themes were validated by comparison of findings by the three reviewers and triangulation of findings from interviews and classroom observations.

Findings

The teachers' thinking about mathematics, word problems, problem solving and the teaching and learning of word problems played a significant role in framing their teaching of word problems in a classroom context. For all of the participants, for the most part, there was consistency between their thinking about teaching word problems and what they actually did in the classroom. There was evidence of some of them, over their careers, having consistently worked at resolving significant conflicts they were aware of between their beliefs about mathematics and their teaching that could help to explain this consistency (Chapman, in press).

The thinking of the participants identified as exemplary teachers reflected a broader range of characteristics and more depth in conceptualizing word problems and teaching word problems than that of the traditional teachers who were participants in the larger study. Each participant was unique in her/his thinking and behavior and there were differences in how each characterized her/his conceptions and in her/his classroom actions. However, as exemplary teachers, there were many similarities conceptually resulting from a common underlying theme in their thinking and behavior that reflected a humanistic view of mathematics, word problems and problem solving and a student-centered view of learning and teaching. All of the teachers thought that,

and a student-centered view of learning and teaching. All of the teachers thought that, as one of them put it, "it does not work to just stand up in front and say, okay, if you see a question like this you do this, this, and this, and this is how you solve it." They also acknowledged that the students who came into their classes generally had a fear of word problems, but they believed that this fear could be significantly reduced if the situation was set up to facilitate this. Thus their teaching was characterized by several factors that were intended to enhance students' affective relationship to word problems and cognitive perspective of dealing with word problems. A detailed discussion of all the key aspects of the teachers' thinking and behaviors cannot be adequately provided here, so the focus will be on outlining some of the key factors which reflect what the teachers valued in framing how they created and facilitated effective/meaningful instruction in the teaching of word problems.

Although other factors (e.g., conceptions of mathematics, word problems, and problem solving) played a role, the teachers considered the students as the primary basis of determining what their teaching of word problems should look like. Their student-centered approach to teaching was directly related to the importance they placed on the following five student characteristics they intentionally incorporated or tried to achieve in their teaching: (i) Real life activities/experiences: This was used to make the word problems relevant or interesting for students. (ii) Ability: This was used to determine how and when to integrate difficult problems with easier ones. (iii) Affective factors, e.g., fear, motivation, sense of success, confidence: This formed a basis for the teachers' goal to get students motivated to move from fear to confidence and success. As one teacher explained: "The teacher should also help students develop a feeling of success, to experience success and to be confident in their ability to do word problems and to see them as fun." (iv) Autonomy: This was used to give students ownership of their learning, e.g., selecting word problems and solution processes that they found to be meaningful. (v) Understanding: This was interpreted as allowing students to construct their own meaning, e.g., their interpretations of word problems and solution processes. These characteristics are not elaborated on here, but are reflected in the description of the teaching characteristics that follow.

The exemplary teachers' thinking and classroom behaviors revealed eight characteristics they valued that formed the essence of their teaching of word problems. While they are all listed here, they are described with varying depth that is reflective of the constraint on space and not importance of each characteristic. All quotes in the description of each characteristic are from the data.

(i) Trialogue Relationship: The general structure of the teachers' teaching was a triangular relationship between the teacher, students and word problem, i.e., there was a trialogue in which both teacher and students could learn something from each other. In this model (Chapman, forthcoming), the teacher specifies the problem for him/herself based on his/her interpretation of it and is speci-

imposing this interpretation on students, the teacher allows them, individually and/or as small groups, to specify the problem for themselves based on their interpretations of it and to be specified by what is allowed by the context of the problem. Finally, by listening to, observing and strategically intervening in the students' relationship with the problem, the teacher allows him/herself to be specified by the students and the students to be specified by the teacher in an open-ended way based on the teacher being flexible and open to a range of justifiable possibilities for solution to the problem. *Specified* is being used in a phenomenological context and relates to how a phenomena is experienced or provides options for it to be experienced.

(ii) Connected-Separate Knowing: The teachers often supported students working from a form of *connected-knowing perspective* to a *separate-knowing perspective* (Chapman, forthcoming). In the former, students were allowed to focus on the social context of the problem by trying to experience the problem, to relate it to their personal world. The teachers viewed this as being motivational, so to facilitate it, they preferred to work with problem situations that had "realistic value for the students." When such problems were not readily available, e.g., in case of traditional textbook problems, the teachers encouraged students to make fun of "stupid problems" and joke about the cover story of the problem, to revise problem situations/context, and to construct problems about things/situations that they found interesting to share with each other. For example, as one teacher explained: "If they say 'it's a stupid problem', I say, 'Ok, so how would you write this problem? You write the problem so it has more meaning for you instead of using some name that you don't know in the problem.'" The teachers treated this process of personalizing the problem context as both a way of knowing and a means of motivating students. They viewed it as paving the way for students to move beyond surface features of the problem and to engage in separate knowing, i.e., focusing on the mathematical context of the problem by suppressing the self in relation to the social context of the problem and taking an impersonal stance toward the problem. For example, students were required to reflect on the problem to see the structure of it and how they could generalize it. Students were also required to reflect on the solution to allow them to see the structure of it and the circumstances under which it would work and not work. The goal during separate knowing was on developing and applying some model for dealing with problems of similar mathematical context.

(iii) Integration: The teachers preferred "integrating the word problems throughout the course and not treating them as something separate or as an isolated topic of mathematics" ... "[treating them] as something that is ongoing and not a particular unit in mathematics." For e.g., the teachers would use word prob-

particular unit in mathematics." For e.g., the teachers would use word problems to start, develop and/or end each topic.

(iv) Collaboration: The teachers emphasized collaboration in developing the solution process when word problems/new topics were introduced. This was done through whole-class discussions and small-group work. Whole-class discussion was always accompanied with small-group work, which could occur before and/or after it. During the whole-class discussions, the teachers' focused on "drawing on the students' experiences and perspectives". There were various ways in which the group-whole-class combination unfolded depending on the teacher, students and topic. In one scenario, the teacher would put students in groups at the beginning of the year and give them some word problems "to look at and to see what they come up with on their own". This is followed by a whole-class discussion where the students have to explain and discuss their interpretation of the problem and their solutions, whether completed or not, and reflecting on their approach. As this teacher explained, they have to address, "What did we [the students] do? What tools did we use? Why did it work or not work? Is there a better way?"

Collaboration among students was also encouraged, with individual work, for practicing problems. The teachers viewed collaboration among students as being important in helping them "to understand that there are others who are experiencing the same difficulties as themselves, who can have a different perspective that they can learn from, and they can bring something to the problem situation that others may not have thought of." Thus, in general, the teachers viewed this shared experience as playing an important role in motivating students and giving them confidence to work individually.

(v) Questioning: The teachers used a questioning approach to facilitate whole-class discussions or to intervene in the groups. Questioning was in the form of questions and prompts that required students to engage in mathematical thinking and to share their thinking – "their experiences and perspectives."

(vi) Writing: The teachers gave students opportunities to write as a way of learning how to approach word problems. For example, students got to construct word problems and were encouraged to write about their thinking when they solved a word problem, particularly in situations where they were encountering difficulties getting started or getting to a resolution of the problem. As one teacher explained, when they were stuck, her students were required to write about "how they started the problem and how this didn't work and that didn't work. ... [This allows them]to describe their process so they can see that they are on a journey, if you will, to find the solution and sometimes the journey does not go right but at least they've started and done something. They should begin writing whatever they are thinking of immediately, so that they don't

leave blank pieces of paper and they are not afraid to try different kinds of things."

(vii) Choice/ Flexibility: The teachers gave students the choice to use "whatever mathematical tools they are made aware that they have to solve the problems." As another teacher said, "it doesn't make any difference what kind of methodology they use as long as it is logical and they can explain it and let other people understand it." Another noted, "If we mark them on what they can show us and not just the right answer or the wrong answer, we can develop that confidence in students." Students were also given choices on tests and assignments. For example, with practice problems, some teachers sometimes assigned to students 5 or 6 "longer problems" during the class or for homework, but required that "they only turn in the ones that they think were their two best solutions so that they have some choice." Sometimes on tests, they were also given a choice.

(viii) Assessment Disclosure: The teachers made students "aware of how these problems will be assessed". For example, they helped students to understand how they were going to be graded, "whether it is by use of rubrics or something else". One way this was done was by "looking at examples of solutions that ranged from excellent to really terrible" in terms of how they were presented and not "the correct way" and discussing them from an assessment perspective.

The above eight teaching characteristics were reflected in all of the teachers' thinking and classrooms, but there were variations in how they were articulated and lived in the classroom. Thus, each teacher was unique in how these teaching characteristics unfolded in her/his classroom. For example, each teacher weaved his/her classroom activities in teaching algebraic word problems with systems of linear equations with his/her own personal touch. The following is an example of one of the teachers' approach for a unit on systems of equations. The whole unit, as opposed to a single class, is presented because of the way the word problems are integrated throughout the unit as an integral part of it. It is also written in a way to reflect the teacher's ongoing use of the approach.

One week prior to starting and discussing the topic in class, the teacher asks the students to collect pictures of graphs that intersect from any source other than a mathematics textbook. The graphs do not necessarily have to be lines. They can be any graphs that intersect. But they should represent actual real-life situations. These pictures are considered to be related to word problems because there is a cover story associated with them.

For the first class of the unit on systems of linear equations, students bring their pictures and share them in a whole-class setting. They spend the whole class looking at

and discussing everyone's picture. Each student shows his/her picture and talks about what the graph represents, the significance of the graph, what it means when graphs intersect, what the intersection shows, why the intersection is important, why anyone would want to find the intersection in the first place. The graphs the students collect are usually about business/economics and physical science situations. Economics situations tend to be dominant because, according to the teacher, they are talking about money with which the students seem to relate more meaningfully. For homework, the teacher gives the students a word problem that requires them to determine, if they have a part-time job, whether it is better to have a fixed hourly rate or a fixed weekly salary plus commission over a specified time period. The students are to try and solve the problem in any way they could.

The second class starts with sharing and discussion of the students' solutions to the homework problem. The teacher poses questions about how the graphical approach was or can be used to solve the problem, how the point of intersection or break-even point was or is useful to the analysis and solution of the problem, and the relevance of solving systems of two linear equations to the students' real-life experiences. The teacher then points out that since they [teacher and students] have decided that it is important to look at where graphs intersect, then they are going to particularly study lines, linear equations, and their intersection. She asks students to return to the graphs they had collected and, for graphs involving two intersecting straight lines, to work in groups to set up the equations for each set of lines using data they read off the graphs. Following this, the teacher facilitates a whole-class discussion of the meaning of the algebraic representations in the context of the applications and why they are useful. In particular, the class discusses the usefulness of an algebraic approach instead of a graphical approach to find the point of intersection. The students, working in groups, then use their graphing calculators to plot examples of special cases of systems of two linear equations that the teacher writes on the chalkboard, e.g.,

$x + y = 4$ and $2x + 2y = 8$

$2x + y = 3$ and $2x + y = 5$.

These are intended for students to investigate and discuss other relationships between two equations, e.g., "what happens if they [the lines] do not intersect? What does it mean if one line is on top of another?" The investigation is followed by whole-class sharing and discussion of their findings. To end the class, the teacher requests that students form groups and each group is assigned a different method to solve systems of equations as required by the curriculum. As she explained, "One group looks at solving these systems of equations graphically, another group looks at solving them using an addition method or a subtraction method [i.e., elimination method] and the third group looks at solving them by substitution."

For the next two classes, students work in their groups investigating the particular solution pattern for the one method to solve a system of two linear equations they are

assigned. The goal is for the group to study solved examples by themselves to try to understand the structure of the method and be able to teach it to others. Thus the students are developing their own versions of what the methods are about. The teacher circulates, observes and intervenes with questions or a counter example to make students think about any limitations she sees in the way they understand the method, e.g., "Would that work for ...?" Students are left to resolve all conflicts for themselves. The teacher assigns exercises to each group to use to test the understanding they construct of the method. The exercise contains equations with coefficients and constants that are all whole numbers, all integers, all rational numbers or some combination of these numbers. For homework, the teacher asks the students to make up a word problem that can be solved with the method they explored in class. The teacher rewards students for creativity, so reproducing problems from a textbook is discouraged and not valued. To end the class, the students have to plan how to teach their approach to others in a way that is interesting and will help them to understand it.

In the fifth class, each group teaches the method they explored to the other groups. This is done by representatives from the teaching group going to each of the other groups to explain to them what they had done in their own group and teach them the method. They are to also make a case that their method is the most efficient. Again, the teacher circulates, observes and intervenes with a counter example to make students think about any limitations she sees in the way they understand the method. Students are left to resolve all conflicts for themselves. For homework, the students are assigned to select and complete practice exercises from those exercises assigned to the groups for each method that have not been completed in class before it ended. The students also are assigned "to bring in real-world word problems that they find elsewhere, other than the textbook, to talk about how they use or can solve systems." For example, "one of the kids brought in internet prices, so they were looking at going with AOL or someone else, and so they brought in a graph that showed these two particular companies and where the break-even point would be so that it wouldn't make any difference which one they had." The teacher assigns 20 percent of their unit mark for "finding a pertinent problem that uses systems and solving it".

In the next two classes, each group works on word problems the teacher selects from the textbook and from what the students have developed. The teacher provides open-ended hints whenever students are stuck but never tells them explicitly how to do anything. Each group then "comes to the front of the class with a different problem that they want to share with the class." Any problems no one in the class is able to do, even after receiving the hints, are identified at the end of the class. Students are asked to continue to think about them for homework.

In the eighth class, the unresolved problems are dealt with in a whole-class discussion in which the teacher poses questions that help the students to arrive at a solution collectively. For the remainder of the class, the teacher facilitates a whole-class discus-

sion on informal extensions of the topic posing questions like, "what if there are three equations instead of two?"

Finally, in the last class of the unit, the teacher gives the students a written test on the topic. For word problems on the test, the teacher includes the final answers. Thus, there is no mark for the final answer, reinforcing the importance of understanding and explaining [through writing] the process.

This teacher's approach is far removed from the traditional classroom approach to dealing with systems of equations, and the most innovative of all of the participants' approaches. But all of the teachers taught in a way that reflects current reform recommendations in mathematics education. These teachers, at the beginning of a semester, explicitly worked at creating the tone in their classroom to allow students to work in this way. In general, their teaching had positive effects on students' attitude and learning of word problems.

Conclusion

The study shows that for these teachers, effective teaching of word problems is more complicated than walking students through examples of solution patterns or heuristics. Instead, it is a particular interplay between teacher, student and word problem that is orchestrated by the teacher based on what the teacher values. The findings provide evidence for teachers that the teaching of high school mathematics does not have to be dominated by telling. But a relativistic view of mathematics teaching based on students' understanding and motivation is desirable and achievable. The findings also reveals specific factors, from real classrooms, of teachers' way of thinking and classroom behaviors that are important to motivate and engage students meaningfully and effectively in doing word problems. This has implications for what we consider and how we work with prospective and experienced teachers in order to help them to develop/change/enhance their teaching of word problems.

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